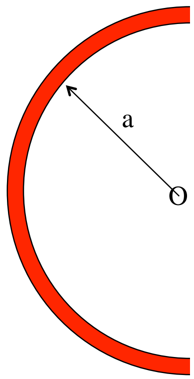
Problem 25.40

A semicircular insulator has charge placed uniformly on it's length. Derive an expression for the electrical potential generated at the semicircle's center.



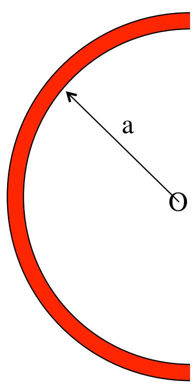
There are two ways to do this. I will show you the easy way first. I'll then show you the more technically sophisticated way (this latter approach would be required if the charge was not uniformly distributed over the surface).

The easy way: The key here is to remember that the electrical potential of a point charge is simply a scalars that is a function of 1/r. As all of the charge pieces is the same distance from the point "O" (I'll call the radius "a"), each tiny piece of charge "dq" on the insulator will create a differential electrical potential equal to

$$dV = k\frac{dq}{a}$$

Summing all the bits of charge up yields:

$$V = k\frac{q_{total}}{a}$$



Noting that:

$$2\pi a = \text{circumference} = 2(.14\ \text{m})$$

$$\Rightarrow \quad a = .14/\pi = 4.456\text{x}10^{-2}\,\text{m}$$

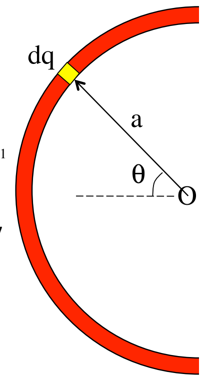
we can write:

$$V = k\frac{q_{total}}{a}$$

$$= \left(9\text{x}10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(-7.5\text{x}10^{-6}\text{C}\right)}{\left(4.456\text{x}10^{-2}\text{m}\right)}$$

$$= -1.5\text{x}10^{6}\,\text{Vw}$$

Option 2: What if the charge isn't uniformly distributed over the surface. What if, for instance, there was some linear charge density function that identified how much charge there was *per unit length* as a function of θ (see sketch), as in $\lambda = k_1\theta$, where k_1 is a constant. What then?



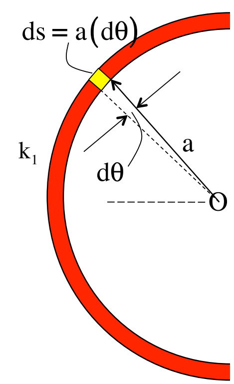
In that case, you would have to identify a differentially small amount of charge *dq*, and determine what electrical potential it would produce at "O," then sum all the differential electrical potentials over the entire rod. To start, then, we write:

$$V = \int dV = k\frac{\int dq}{a}$$

To make this work, we need to find a way to relate "dq" to the physical parameters we have been given.

So how do we get "dq" in terms of parameters we have been given? Look at the sketch. If we assume the differential length upon which "dq" resides is "ds," the "ds" will be related to the radius and $d\theta$ by:

$$ds = a d\theta$$



and the charge "dq" on "ds" would become:

$$dq = \lambda ds = (k_1\theta)(a d\theta)$$

That means the field due to "dq" is:

$$dV = k\frac{\lambda ds}{a}$$

$$= k\frac{(k_1\theta)(a d\theta)}{a}$$

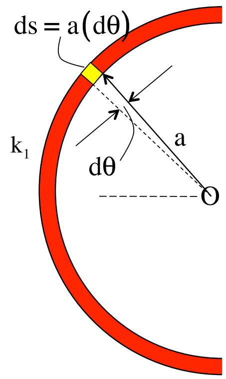
The total field generated by all of the charge becomes twice the field generated by the charge between $\theta = 0$ and $\theta = \frac{\pi}{2}$:

$$V = 2\int_{\text{half the rod}} dV = 2k\frac{\int_{\theta=0}^{\pi/2}(k_1\theta)(a\,d\theta)}{a}$$

$$\Rightarrow \quad V = 2kk_1\frac{\theta^2}{2}\Big|_{\theta=0}^{\pi/2}$$

$$\Rightarrow \quad V = kk_1\left[\left(\frac{\pi}{2}\right)^2 - 0\right]$$

$$\Rightarrow \quad V = \frac{kk_1\pi^2}{4}$$



Not to beat a dead horse, but what if we tried the more formal approach with the original problem. Then λ would be a constant and we could write:

$$V = \int dV = k\frac{\lambda ds}{a}$$

$$= k\int\frac{\lambda(a\,d\theta)}{a}$$

$$= 2k\lambda\int_{\theta=0}^{\pi/2} d\theta$$

$$= 2k\lambda\theta\Big|_{\theta=0}^{\pi/2}$$

$$= \not{2}k\lambda(\pi/\not{2})$$

$$= \left(9\text{x}10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(\frac{-7.5\text{x}10^{-6}\ \text{C}}{(.14\ \text{m})}\right)\pi$$

$$= -1.51\text{x}10^6\ \text{V}$$

Ain't that a kick in the pants?